Cross Modulation and Nonlinear Distortion in RF Transistor Amplifiers*

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Summary—**In order to avoid untractable calculations, the transistor four-pole is assumed to be short-circuited for ac at its output. Furthermore, the internal impedance of the signal source is assumed to be zero. First the nonlinear distortion effects in a grounded base intrinsic transistor are calculated. Then, the formulas are reverted to a grounded emitter intrinsic transistor, taking into account the extrinsic base lead resistance. They are confirmed by measurements of third harmonic distortion and of cross modulation. The measured curves of cross modulation vs collector bias current show a sharp minimum. This unexpected effect is explained by an extension of the theory, which takes into account previously neglected terms. The explanation is successfully tested by experiments. Comparisons with cross modulation in amplifier tubes are made.**

List of Symbols

A = area of the cross sections of the p-n junctions.
A_1, A_2, A_3 = amplitudes of the first, second, and third harmonics in the ac collector current.
B = extrinsic base.
B' = intrinsic base.
C = extrinsic collector.
C' = intrinsic collector.
d_2, d_3 = relative second and third harmonic distortions in the modulation.
D_p = diffusion-constant of holes in n-type semiconductor.
e = amount of electron charge.
E = extrinsic emitter.
E' = intrinsic emitter.
f_1 = frequency of the AF modulation.
f_s = frequency of the spurious signal.
$f_{\alpha b}$ = cut-off frequency of a transistor in grounded base connection.
f_0 = frequency of the desired signal.
i_c = instantaneous value of the collector ac current.
i_e = instantaneous value of the emitter ac current.
$\mathbf{I}_b$ = base ac current in complex notation.
I_B = base dc current.
$\mathbf{I}_c$ = collector ac current in complex notation.
I_C = collector dc current.

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* Manuscript received by the PGED, May 5, 1959.

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$\mathbf{I}_e$ = emitter ac current in complex notation.
I_E = emitter dc current.
$\mathbf{I}_{pc}$ = collector ac hole current in complex notation.
$\mathbf{I}_{pe}$ = emitter ac hole current in complex notation.
$\mathbf{I}_1$ = input ac current of a four-pole in complex notation.
$\mathbf{I}_2$ = output ac current of a four-pole in complex notation.
$j = \sqrt{-1}$.
k = constant of Boltzmann.
L_p = average diffusion length of holes in n-type semiconductor.
M = the degree of amplitude modulation of the spurious signal.
M' = new degree of amplitude modulation after alteration of the degree of amplitude modulation.
M_k = degree of cross modulation.
p = density of holes.
p_n = thermodynamic equilibrium density of holes in n-type semiconductor.
$R_{bb'}$ = base lead resistance of the transistor.
R_{bz} = extra resistance in the base lead.
R_{ez} = extra resistance in the emitter lead.
t = time.
T = temperature in degrees Kelvin.
v = instantaneous value of ac voltage.
$v_{e'b'}$ = instantaneous value of intrinsic emitter-base ac voltage.
v_s = instantaneous value of the spurious signal voltage.
V_m = amplitude of ac voltage.
$V_{e'b'm}$ = amplitude of the intrinsic emitter-base ac voltage.
$V_{C'B'}$ = intrinsic collector-base dc voltage.
V_{CE} = collector-emitter dc voltage.
$\mathbf{V}_{eb}$ = emitter-base ac voltage in complex notation.
$\mathbf{V}_{e'b'}$ = intrinsic emitter-base ac voltage in complex notation.
V_{ebm} = amplitude of the emitter-base ac voltage.
V_{eb} = rms value of the emitter-base ac voltage.
$V_{E'B'}$ = intrinsic emitter-base dc voltage.
V_{sm} = amplitude of the spurious signal.
V_s = rms value of the spurious signal.
V_{0m} = amplitude of the desired signal.
V_0 = rms value of the desired signal.
$\mathbf{V}_1$ = input ac voltage of a four-pole in complex notation.
$\mathbf{V}_2$ = output ac voltage of a four-pole in complex notation.
w = the effective base width.
x_{nc} = boundary of the collector-base depletion layer on the base side in a p-n-p transistor.
x_{ne} = boundary of the emitter-base depletion layer on the base side in a p-n-p transistor.
$\mathbf{Y}_{11}, \mathbf{Y}_{21}$ = admittance parameters of a four-pole.
$\mathbf{Y}'_{11e}$ = input admittance of the intrinsic transistor in grounded emitter connection.
γ = emitter efficiency.
$\Lambda = e/kT$.
ω = angular frequency.
ω_1 = angular frequency of the AF modulation.
ω_s = angular frequency of the spurious signal.
ω_0 = angular frequency of the desired signal.
τ_p = average life time of holes in n-type semiconductor.

INTRODUCTION

TRANSISTOR amplifiers are very often treated as four-poles. Therefore, in considering the nonlinear distortion effects of transistor amplifiers, it is appropriate to start from the more general case of nonlinear four-poles.

The complete treatment of nonlinear four-poles with parameters, which are independent of frequency, is straightforward.[1,2] With nonlinear four-poles, in which the parameters are dependent on frequency, the calculations are much more intricate. The reason for this is that phase angles, which are different from 0 or π, may occur in the latter case, but not in the former one. The difficulties of a general mathematical treatment of the latter case are so formidable, that the present authors will restrict the calculations to the case of a four-pole with short-circuited output. Under this restriction the four-pole equations with admittance parameters are reduced to

$$\mathbf{I}_1 = \mathbf{Y}_{11}\mathbf{V}_1, \tag{1a}$$

and

$$\mathbf{I}_2 = \mathbf{Y}_{21}\mathbf{V}_1 \tag{1b}$$

Eq. (1b) is of special interest, as it describes a relationship between an input and an output entity and hence a transfer characteristic of the four-pole. As a further simplification, it is assumed that the signal source at the input has zero internal resistance. Only by this assumption is it possible to choose the input signal voltage independently of the properties of the four-pole. This assumption, as well as the further ones mentioned above, are adhered to in the further text of this paper.

I. CALCULATION OF NONLINEAR DISTORTION WITH AN INTRINSIC TRANSISTOR IN GROUNDED BASE CONNECTION

The calculations are based on a p-n-p junction transistor, as considered originally by W. Shockley. Carrier motion is then caused by diffusion only. Current densities are assumed to be so small that no electric field effects arise. Ohmic voltage drops outside the depletion layer are

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[1] N. I. Meyer, "Nonlinear Distortion in Transistor Amplifiers at Low Signal Levels and Low Frequencies," IEE Monograph No. 209 R; November, 1956.

[2] G. A. Spescha and M. J. O. Strutt, "Theoretische und experimentelle Untersuchung der Verzerrungen in Niederfrequenz-Flächen-transistor-Vierploen," *Arch. elekt. Ubertregung*, vol. 11, pp. 307–320; March, 1957.

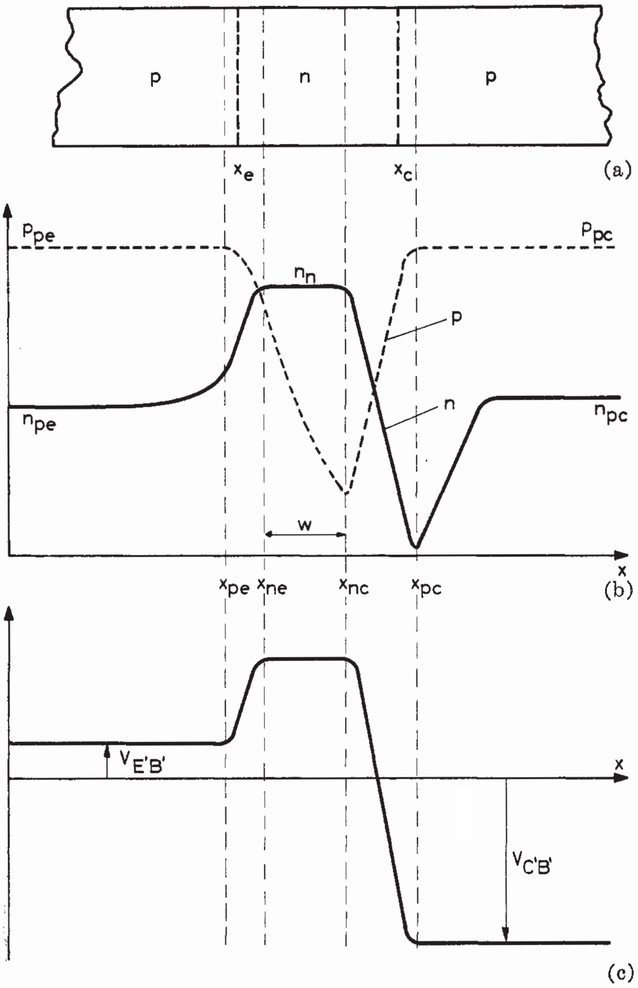

Fig. 1—Simplified picture of a *p-n-p* transistor under normal operating conditions. (a) Geometry. The technological junctions emitter-base and base-collector are indicated by x_e and x_c respectively. (b) Approximate curves of densities of electrons (n) and holes (p). The emitter-base depletion layer is bounded by x_{pe} and x_{ne} respectively. The base-collector depletion layer is bounded by x_{nc} and x_{pc} respectively. The effective base width is w. (c) Approximate voltage distribution along the transistor axis. This picture does not take account of the acceptor and donor profiles obtained by the application of special alloying or diffusion processes in manufacturing the transistor.

neglected. Hence, the external voltages are concentrated across the depletion layers.

In this section an intrinsic transistor in grounded base connection is considered. The resulting expressions are extended to the grounded emitter connection in a subsequent section. Thereupon, the extrinsic elements are taken into account. Thus, approximate distortion equations are obtained for a transistor including extrinsic elements in the grounded emitter connection. This represents the most important case for practical applications. The nomenclature "intrinsic transistor" and "extrinsic elements" as used in this paper is in complete agreement with the usual meaning of these expressions in IRE papers of recent years.

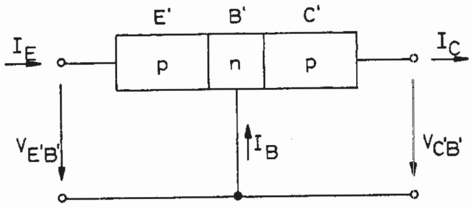

Fig. 2—*p-n-p* transistor in grounded base connection, with indication of our choice of positive directions of currents and voltages.

In all calculations a short circuit connection with respect to ac is assumed at the transistor output. In the grounded base connection, this means an ac short circuit between collector and base. For the solution of the differential equation describing diffusion, the boundary equations contain series expansions. In the usual treatment of this problem, the expansions are cut off after the second term. Here, however, further terms are taken into account. The number of terms to be taken into account in the series expansions depends on the distortions under consideration. For practical purposes it will often be satisfactory if the third harmonic is still taken into account, leaving out the fourth and higher ones. In this case, the fourth and higher powers in the series expansion may be neglected. Of course, these neglected terms also contribute to the harmonic distortions under consideration. However, accounting for these higher terms would encumber the calculations very much. The measurements to be discussed later show that the neglections mentioned are permissible for a relatively large range of signal amplitudes. The solution itself is extended correspondingly. The carrier density in the base region is calculated as dependent on the linear dimensions and on time. In the calculation of harmonic current components, a sine voltage is assumed between emitter and base. Some symbols are shown in Fig. 1. The choice of positive directions of voltages and currents in the grounded base connections of a *p-n-p* junction transistor are shown in Fig. 2. These directions are valid for dc and ac quantities in the grounded base connection. We shall use the abbreviation $\Lambda = e/kT$. If $V_{e'b'm} \ll 1/\Lambda$, we obtain for the boundary condition of the holes at $x = x_{ne}$, the value $p = p(x_{ne})$ (see also Fig. 1; the derivation of this equation is shown in the Appendix):

$$
\begin{aligned}
p = p(x_{ne}) = Re\Big\{ p_n \exp(\Lambda V_{E'B'}) \Big[1 &+ \frac{1}{2} \frac{\Lambda^2 V_{e'b'm}^2}{2!} \\
&+ \Lambda V_{e'b'm} \left(1 + \frac{3}{4} \frac{\Lambda^2 V_{e'b'm}^2}{3!} \right) \exp(j\omega t) \\
&+ \frac{1}{2} \frac{\Lambda^2 V_{e'b'm}^2}{2!} \exp(j2\omega t) \\
&+ \frac{1}{4} \frac{\Lambda^3 V_{e'b'm}^3}{3!} \exp(j3\omega t) \Big] \Big\}. \qquad (2)
\end{aligned}
$$

At $x = x_n{}^2$ (see Fig. 3):

$$p = p(x_{nc}) = p_n \exp(\Lambda V_{C'B'}). \qquad (3)$$

In the latter case no ac components arise, as an ac short circuit is presupposed between collector and base.

The solution of the diffusion equation

$$\frac{p(x,\,t)}{\partial t} = \frac{p_n - p(x,\,t)}{\tau_p} + D_p\,\frac{\partial^2 p(x,\,t)}{\partial x^2} \tag{4}$$

is assumed to be as follows:

$$\begin{aligned}\mathbf{p}(x,\,t) &= P_0(x) + \mathbf{P}_1(x)\exp(j\omega t)\\ &\quad + \mathbf{P}_2(x)\exp(j2\omega t) + \mathbf{P}_3(x)\exp(j3\omega t).\end{aligned} \tag{5}$$

This solution is inserted into the diffusion equation. Thereby, differential equations are obtained for $\mathbf{P}_1(x)$, $\mathbf{P}_2(x)$ and $\mathbf{P}_3(x)$, from which these quantities may be calculated, taking account of the boundary conditions (2) and (3). From the resulting hole density, the hole currents at the emitter and at the collector may be obtained. The ac components of these currents are

$$\begin{aligned}\mathbf{I}_{pe} &= \frac{e\,D_pAp_n}{L_p}\exp(\Lambda V_{E'B'})\\ &\cdot\left\{\frac{\sqrt{1+j\omega\tau_p}}{\tanh\left[\frac{\sqrt{1+j\omega\tau_p}}{L_p}\,w\right]}\Lambda V_{e'b'm}\right.\\ &\cdot\left(1+\frac{3}{4}\,\frac{\Lambda^2V_{e'b'm}^2}{3!}\right)\exp(j\omega t)\\ &+\frac{\sqrt{1+j2\omega\tau_p}}{\tanh\left[\frac{\sqrt{1+j2\omega\tau_p}}{L_p}\,w\right]}\,\frac{1}{2}\,\frac{\Lambda^2V_{e'b'm}^2}{2!}\exp(j2\omega t)\\ &\left.+\frac{\sqrt{1+j3\omega\tau_p}}{\tanh\left[\frac{\sqrt{1+j3\omega\tau_p}}{L_p}\,w\right]}\,\frac{1}{4}\,\frac{\Lambda^3V_{e'b'm}^3}{3!}\exp(j3\omega t)\right\},\end{aligned} \tag{6}$$

$$\begin{aligned}\mathbf{I}_{pc} &= \frac{e\,D_pAp_n}{L_p}\exp(\Lambda V_{E'B'})\\ &\cdot\left\{\frac{\sqrt{1+j\omega\tau_p}}{\sinh\left[\frac{\sqrt{1+j\omega\tau_p}}{L_p}\,w\right]}\Lambda V_{e'b'm}\right.\\ &\cdot\left(1+\frac{3}{4}\,\frac{\Lambda^2V_{e'b'm}^2}{3!}\right)\exp(j\omega t)\\ &+\frac{\sqrt{1+j2\omega\tau_p}}{\sinh\left[\frac{\sqrt{1+j2\omega\tau_p}}{L_p}\,w\right]}\,\frac{1}{2}\,\frac{\Lambda^2V_{e'b'm}^2}{2!}\exp(j2\omega t)\\ &\left.+\frac{\sqrt{1+j3\omega\tau_p}}{\sinh\left[\frac{\sqrt{1+j3\omega\tau_p}}{L_p}\,w\right]}\,\frac{1}{4}\,\frac{\Lambda^3V_{e'b'm}^3}{3!}\exp(j3\omega t)\right\}.\end{aligned} \tag{7}$$

Making use of well-known equations for the dc current components, we obtain the following simplification:

$$\frac{e\,D_pAp_n}{L_p}\exp(\Lambda V_{E'B'}) = (I_{PE} - I_{PES})\tanh\frac{w}{L_p},$$

which may be inserted into (6) and (7). With most transistors we may assume that

1) $\frac{w}{L_p} \ll 1$, resulting in: $\tanh\frac{w}{L_p} \cong \frac{w}{L_p}$,

2) $I_{PE} \gg I_{PES}$ resulting in: $I_{PE} - I_{PES} \cong I_{PE}$,

3) $I_{PE} \gg I_{NE}$ resulting in: $I_E \cong I_{PE}$.

The latter condition means that electron currents are neglected relative to the hole currents and that the emitter efficiency γ is equal to unity.

If we apply these calculations to the grounded emitter case, it is necessary to replace 3) by $I_{PE} = \gamma I_E$ in the evaluation of the ac collector current.

If the frequencies under consideration are small relative to the cutoff frequencies in the grounded base connection, we have

4) $|\,n\omega\tau_p\,| \ll 1$

5) $\left|\,n\omega\,\frac{w^2}{2\,D_p}\,\right| \ll 1.$

Using the simplifications 1) through 5) the following ac current expressions are obtained:

$$\begin{aligned}\mathbf{I}_e \cong I_E\Bigg[&\left(1+j\omega\,\frac{w^2}{3\,D_p}\right)\Lambda V_{e'b'm}\\ &\cdot\left(1+\frac{3}{4}\,\frac{\Lambda^2V_{e'b'm}^2}{3!}\right)\exp(j\omega t)\\ &+\left(1+j2\omega\,\frac{w^2}{3\,D_p}\right)\frac{1}{2}\,\frac{\Lambda^2V_{e'b'm}^2}{2!}\exp(j2\omega t)\\ &+\left(1+j3\omega\,\frac{w^2}{3\,D_p}\right)\frac{1}{4}\,\frac{\Lambda^3V_{e'b'm}^3}{3!}\exp(j3\omega t)\Bigg],\end{aligned} \tag{8}$$

$$\begin{aligned}\mathbf{I}_c \cong \gamma I_E\Bigg[&\left(1-j\omega\,\frac{w^2}{6\,D_p}\right)\Lambda V_{e'b'm}\\ &\cdot\left(1+\frac{3}{4}\,\frac{\Lambda^2V_{e'b'm}^2}{3!}\right)\exp(j\omega t)\\ &+\left(1-j2\omega\,\frac{w^2}{6\,D_p}\right)\frac{1}{2}\,\frac{\Lambda^2V_{e'b'm}^2}{2!}\exp(j2\omega t)\\ &+\left(1-j3\omega\,\frac{w^2}{6\,D_p}\right)\frac{1}{4}\,\frac{\Lambda^3V_{e'b'm}^3}{3!}\exp(j3\omega t)\Bigg].\end{aligned} \tag{9}$$

Taking account of the above condition 5), the imaginary components in these expressions, which are independent of time t, may be neglected.

It is possible to calculate higher harmonic current components by suitably extending the boundary condition (2) and the solution (5). We now turn to the calculation of cross modulation. A nonmodulated and an amplitude-modulated signal are assumed between emitter and base:

$$v_{e'b'} = V_{0m}\cos\omega_0 t + V_{sm}(1 + M\cos\omega_1 t)\cos\omega_s t. \tag{10}$$

Here ω_0 is the angular frequency of the desired signal, and ω_s is the angular frequency of the spurious signal. The angular frequency of the modulation of the latter signal is ω_1. This signal $v_{e'b'}$ is inserted into the boundary condition at $x = x_{ne}$. We shall assume that the output of the transistor is tuned to the angular frequency ω_0. Thus, only terms of angular frequency ω_0 and of frequencies in its immediate vicinity are taken into account. Then

$$
\begin{aligned}
p = p(x_{ne}) = Re\Big\{p_n \exp(\Lambda V_{E'B'}) \\
\cdot\Big\{1 + \frac{\Lambda^2}{2!}\frac{1}{2}(V_{0m}^2 + V_{sm}^2) + V_{0m}\exp(j\omega_0 t) \\
+ \frac{3}{2}\frac{\Lambda^3}{3!} V_{0m}V_{sm}^2 M\Big[\exp j(\omega_0 - \omega_1)t \\
+ \exp j(\omega_0 + \omega_1)t\Big]\Big\}\Big\}. \qquad (11)
\end{aligned}
$$

The boundary condition at $x = x_{nc}$ is the same as in (3).

Using these boundary conditions and an appropriate assumed solution and taking into account the simplifications and neglections mentioned above, we obtain the following expressions for the ac collector and emitter currents:

$$
\begin{aligned}
\mathbf{I}_e \cong I_E\Big\{\Big(1 + j\omega_0 \frac{w^2}{3\,D_p}\Big)\Lambda V_{0m}\exp(j\omega_0 t) \\
+ \Big[1 + j(\omega_0 - \omega_1)\frac{w^2}{3\,D_p}\Big] \\
\cdot\frac{3}{2}\frac{\Lambda^3}{3!} V_{0m}V_{sm}^2 M \exp[j(\omega_0 - \omega_1)t] \\
+ \Big[1 + j(\omega_0 + \omega_1)\frac{w^2}{3\,D_p}\Big] \\
\cdot\frac{3}{2}\frac{\Lambda^3}{3!} V_{0m}V_{sm}^2 M \exp[j(\omega_0 + \omega_1)t]\Big\}, \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
I_c \cong \gamma I_E\Big\{\Big(1 - j\omega_0 \frac{w^2}{6\,D_p}\Big)\Lambda V_{0m}\exp(j\omega_0 t) \\
+ \Big[1 - j(\omega_0 - \omega_1)\frac{w^2}{6\,D_p}\Big] \\
\cdot\frac{3}{2}\frac{\Lambda^3}{3!} V_{0m}V_{sm}^2 M \exp[j(\omega_0 - \omega_1)t] \\
+ \Big[1 - j(\omega_0 + \omega_1)\frac{w^2}{6\,D_p}\Big] \\
\cdot\frac{3}{2}\frac{\Lambda^3}{3!} V_{0m}V_{sm}^2 M \exp[j(\omega_0 + \omega_1)t]\Big\}. \qquad (13)
\end{aligned}
$$

In these expressions, the imaginary terms, which are independent of time t, may again be neglected, as was mentioned above. Eqs. (12) and (13) yield the desired cross modulation. This will be shown in Section II, dealing with the grounded emitter connection.

Besides the nonlinear distortions which were treated above, some further nonlinear effects arise in the case of amplitude-modulated signals. These effects are distortion of modulation and alteration of the degree of amplitude modulation. In order to calculate these effects, only an amplitude-modulated signal is assumed between emitter and base:

$$
v = V_m(1 + M\cos\omega_1 t)\cos\omega t. \qquad (14)
$$

This signal is inserted into the boundary condition at $x = x_{ne}$. Then, only terms of angular frequency ω and of frequencies in its immediate vicinity are taken into account. This time, however, multiples of the angular modulation frequency ω_1 are not neglected. Assuming a suitable solution of the diffusion equation we finally obtain the following ac current expressions:

$$
\begin{aligned}
i_e = I_E\Lambda V_m\Big\{1 + \frac{3}{4}\frac{\Lambda^2}{3!}V_m^2\Big(1 + \frac{3}{2}M^2\Big) \\
+ M\Big[1 + \frac{9}{4}\frac{\Lambda^2}{3!}V_m^2\Big(1 + \frac{M^2}{4}\Big)\Big]\cos\omega_1 t \\
+ \frac{9}{8}\frac{\Lambda^2}{3!}V_m^2 M^2\cos 2\omega_1 t \\
+ \frac{3}{16}\frac{\Lambda^2}{3!}V_m^2 M^3\cos 3\omega_1 t\Big\}\cos\omega t, \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
i_c = \gamma I_e\Lambda V_m\Big\{1 + \frac{3}{4}\frac{\Lambda^2}{3!}V_m^2\Big(1 + \frac{3}{2}M^2\Big) \\
+ M\Big[1 + \frac{9}{4}\frac{\Lambda^2}{3!}V_m^2\Big(1 + \frac{M^2}{4}\Big)\Big]\cos\omega_1 t \\
+ \frac{9}{8}\frac{\Lambda^2}{3!}V_m^2 M^2\cos 2\omega t \\
+ \frac{3}{16}\frac{\Lambda^2}{3!}V_m^2 M^3\cos 3\omega_1 t\Big\}\cos\omega t. \qquad (16)
\end{aligned}
$$

These expressions (15) and (16) yield the nonlinear effects mentioned above. They will be discussed further in Section II.

II. Calculation of Distortion Effects of an Intrinsic *p-n-p* Junction Transistor in Grounded Emitter Connection.

In Fig. 3, the grounded base and the grounded emitter connections of a transistor have been drawn, showing the chosen positive directions of ac voltages and currents. As compared with the grounded base connection, only the voltage between emitter and base alters its direction in the grounded emitter connection. If the ac expressions of Section I are altered accordingly, we obtain the ac

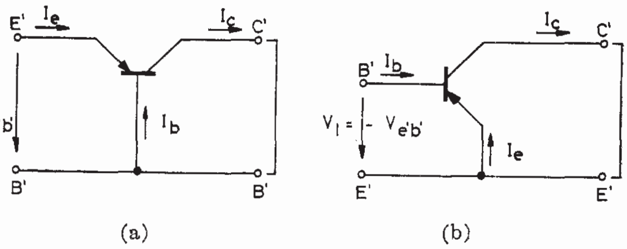

Fig. 3—Grounded base connection (a) and grounded emitter connection (b) of a *p-n-p* junction transistor with indications of our choices of positive directions of alternating currents and voltages.

expressions for an intrinsic transistor in grounded emitter connection. The condition of Section I, stating that an ac short circuit connection exists between collector and base, must also be satisfied in the present case. It is not sufficient to adopt an ac short circuit connection between collector and emitter (the output of the grounded emitter connection). In order to obtain an ac short circuit between collector and base, we must now also adopt an ac short circuit between base and emitter. This means that the internal resistance of the signal source, connected to the input of the four-pole, must be zero. This has already been stated in the introduction and holds for all our calculations. In the further calculations, only the ac collector current is considered, as it represents the desired output entity as dependent on the input voltage.

After some calculations, one finally obtains the following expressions for the relative values of the harmonic distortion:

$$\left|\frac{A_2}{A_1}\right| = \frac{\Lambda}{4} V_{e'b'm} \tag{17}$$

$$\left|\frac{A_3}{A_1}\right| = \frac{\Lambda^2}{24} V^2_{e'b'm}. \tag{18}$$

The degree of cross modulation is approximately given by

$$M_K = \frac{\Lambda^2 M}{2} V^2_{sm}. \tag{19}$$

The relative harmonic distortions of the modulation are expressed by the following definition:

$$d_n = \frac{\text{Amplitude of modulation with angular frequency } n\omega_1}{\text{Amplitude of modulation with angular frequency } \omega_1}.$$

As has already been stated in Section I, small amplitudes V_m are characterized by the condition: $\Lambda V_m \ll 1$.

At small signal amplitudes, one obtains approximately

$$d_2 = \frac{3}{16} \Lambda^2 M V^2_m \tag{20}$$

$$d_3 = \frac{1}{32} \Lambda^2 M^2 V^2_m. \tag{21}$$

At small signal amplitudes and at degrees M of modulation small with respect to unity we obtain

$$\frac{M' - M}{M} = \frac{\Lambda^2}{4} V^2_m. \tag{22}$$

From (17) through (22) the following inferences may be made:

a) The distortions are independent of the dc bias values. This is due to the fact that an intrinsic transistor, in which the currents depend exponentially on the voltages, is considered.

b) The distortions are independent of the particular transistor under consideration and independent of the semiconductor material. The above expressions contain only the signal voltages, the degree of modulation and Λ. This latter quantity contains only constants of nature (e, k) and the absolute temperature T. Hence, the expressions hold for p-n-p or n-p-n transistors, made of germanium or silicon, provided that the underlying assumptions of the above calculations are satisfied.

The statements a) and b) apply only to intrinsic transistors. As soon as extrinsic elements are taken into account they are no longer applicable.

III. CHANGES OF THE DISTORTION EXPRESSIONS, CAUSED BY EXTRINSIC ELEMENTS.

The extrinsic elements are considered in connection with the grounded emitter circuit. The most important extrinsic element is represented by the base resistance.[3–5] The emitter and collector series resistance, the collector leakage resistance and the capacities of the depletion layers are of little consequence in distortion calculations and may hence be neglected. The capacity of the depletion layer between the collector and the base cannot be neglected under most circumstances.[4,5] Here, however, an ac short circuit is always assumed between collector and base. Under this condition, the said capacity is negligible.

Fig. 4 shows the transistor under consideration, including the extrinsic resistance $R_{bb'}$, in a grounded emitter circuit. Hitherto, the distortion was calculated, assuming an intrinsic emitter-base voltage $\mathbf{V}_{e'b'}$. If the extrinsic resistance $R_{bb'}$ is taken into account, this voltage must be replaced by $\mathbf{V}_{eb}$. The calculations are cumbersome, as the former voltage $\mathbf{V}_{e'b'}$ is no longer a simple sine-function of time if the latter voltage is. This distortion effect will, however, be neglected. We shall assume, that both voltages are still sine-functions of time. The voltage $\mathbf{V}_{eb}$ is divided between the resistance $R_{bb'}$ and the input impedance $1/\mathbf{Y}'_{11e}$ of the intrinsic transistor at short-circuited output. Hence

$$V_{e'b'm} = \frac{V_{ebm}}{1 + R_{bb'} \mid \mathbf{Y}'_{11e} \mid}. \tag{23}$$

By (23) the distortions (17) through (22) of the intrinsic transistor may be converted to expressions for the transistor including $R_{bb'}$ in a very simple way:

$$\left|\frac{A_2}{A_1}\right| = \frac{\Lambda}{4} \frac{V_{ebm}}{1 + R_{bb'} \mid \mathbf{Y}'_{11e} \mid} \tag{24}$$

$$\left|\frac{A_3}{A_1}\right| = \frac{\Lambda^2}{24} \frac{V^2_{ebm}}{(1 + R_{bb'} \mid \mathbf{Y}'_{11e} \mid)^2} \tag{25}$$

$$M_K = \frac{\Lambda^2 M}{2} \frac{V^2_{sm}}{(1 + R_{bb'} \mid \mathbf{Y}'_{11e} \mid)^2} \tag{26}$$

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[3] J. M. Early, "Design theory of junction transistors," *Bell Sys. Tech. J.*, vol. 32, pp. 1271–1312; November, 1953.

[4] D. Dewitt and A. L. Rossoff, "Transistor Electronics," McGraw-Hill Book Co., Inc., New York, N. Y.; 1957.

[5] L. J. Giacoletto, "Study of p-n-p alloy junction transistors from d-c through medium frequencies," *RCA Rev.*, vol. 15, pp. 506–562; December, 1954.

$$d_2 = \frac{3}{16}\Lambda^2 M \frac{V_m^2}{(1 + R_{bb'}\,|\,\mathbf{Y}'_{11e}\,|)^2} \tag{27}$$

$$d_3 = \frac{1}{32}\Lambda^2 M^2 \frac{V_m^2}{(1 + R_{bb'}\,|\,\mathbf{Y}'_{11e}\,|)^2} \tag{28}$$

$$\frac{M' - M}{M} = \frac{\Lambda^2}{4}\frac{V_m^2}{(1 + R_{bb'}\,|\,\mathbf{Y}'_{11e}\,|)^2}. \tag{29}$$

By comparing (25) through (29), it is seen that the distortion effects are very akin, as is also the case for the corresponding effects of electron tubes[6–8]. With transistors, however $\mathbf{Y}'_{11e}$ is dependent on frequency. This is to be taken into account in applying the above expressions.

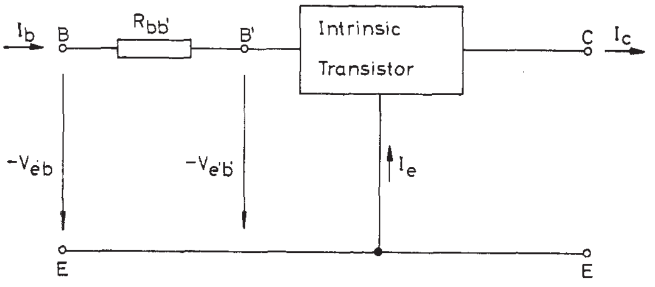

Fig. 4—Intrinsic *p-n-p* junction transistor including extrinsic resistance $R_{bb'}$ in grounded emitter connection.

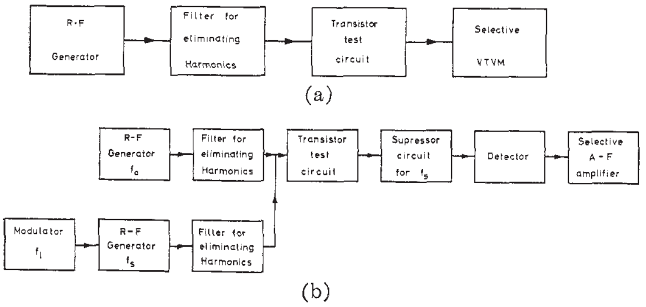

Fig. 5—(a) Measuring set-up for the determination of harmonic distortion. (b) Measuring set-up for the determination of cross modulation.

IV. Experimental Investigation of Distortion Effects.

The theoretical formulas have been tested experimentally by measurements of the third harmonic and of the cross modulation. The measuring arrangements are shown in Fig. 5 as a block diagram. The filter circuits for the harmonic frequencies consist of simple resonant circuits. The transistor circuit is built so that the effective internal resistance of the signal source is smaller than 10 ohms. Between collector and emitter an effective resistance of 100 ohms is connected. This sufficiently satisfies the short-circuited condition. The suppression of the spurious signal at the output was effected by means of a special suppressor circuit.[9] The degree of cross modulation was determined by the use of a substitution method. A signal source of known modulation characteristic was connected at the transistor output. Then the AF signal at the AF output was measured and the modulation of the signal source was adjusted, so as to yield an equal AF output signal as previously with the cross modulated signal. The thesis of the first author contains more elaborate circuit diagrams and descriptions of the measurements.[10]

In order to effect the comparison of the calculated and measured values, the extrinsic resistance $R_{bb'}$ and the intrinsic input admittance $\mathbf{Y}'_{11e}$ must be known. They may be determined experimentally by known methods.[5,11,12] The value of $\mathbf{Y}'_{11e}$ was not measured at each dc bias value, but was calculated according to L. J. Giacoletto[5] using the dc transistor equations.

The measurements were carried out using three germanium *p-n-p* junction transistors. They differ mainly with respect to their cutoff frequenices in the grounded base connection: type 2 N 114, $f_{\alpha b} = 15.2$ mc; type 2 N 113; $f_{\alpha b} =, 9.4$ mc; type 2 *N* 137, $f_{\alpha b} = 6.6$ mc.

In Fig. 6 the calculated and the measured values of the third harmonic as dependent on the collector dc current are shown for the three transistor types mentioned. Considering the several simplifications and neglections incorporated in the formulas, the coincidence between theory and experiment may be called satisfactory. The different values of $R_{bb'}$ mentioned in Fig. 6 are obtained by the insertion of an extra resistance of 100 ohms into the base lead.

Fig. 7 shows the dependence of the third harmonic on the collector current for transistors with different cutoff frequencies. If this cutoff frequency is smaller, the amount of $\mathbf{Y}'_{11e}$ is larger. Hence, according to (23), the transistor input signal corresponding to a given value of third harmonic is larger. This is clearly seen from the curves of Fig. 7.

Fig. 8 shows the relative third harmonic values as dependent on input signal amplitude. It is seen that the third harmonics are proportional to the square of the input signal amplitude within a large range of the latter. This affords a good confirmation of the theory.

Fig. 9 shows the measured and the calculated cross modulation curves as dependent on dc collector current.

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[6] B. G. Dammers, J. Haantjes, S. Otte, and W. H. van Suchtelen, "Application of Electronic Valve in Radio Receivers and Amplifiers," Philips Tech. Library, Eindhoven, The Netherlands, Bk. IV; 1949.

[7] S. Ballantine and H. A. Snow, "Reduction of distortion and crosstalk in radio receivers by mean of variable-mu tetrodes," Proc. IRE, vol. 18, pp. 2102–2127; December, 1930.

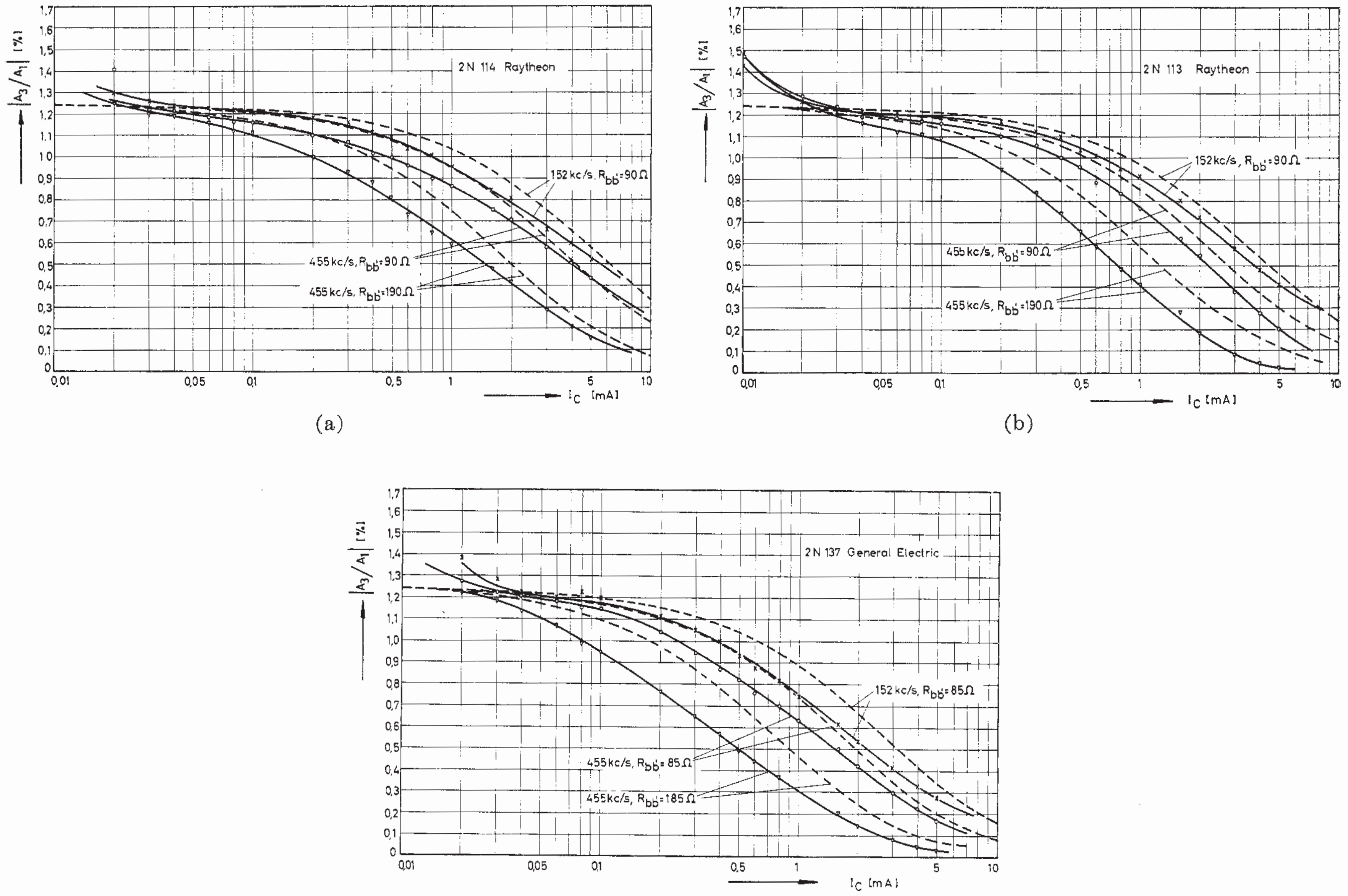
[8] C. S. Bull, "Non linear valve characteristics, a brief discussion on their use," *Wireless Engr.*, vol. 10, pp. 83–88; February, 1933.

[9] M. J. O. Strutt, "Verstärker und Empfänger," Springer Verlag, Berlin, Ger., 2nd ed., 1950.

[10] M. Akgün, "Beiträge zur Kenntnis der nichtlinearen Verzerrungen in Hochfrequenz-Verstärkerstufen mit Transistoren für kleine Signalamplituden," Ph.D. dissertation, Swiss Fed. Inst. of Tech., Zürich, Switzerland; Prom. no. 2904; 1959.

[11] L. J. Giacoletto, "Performance of a radio-frequency alloy junction transistor in different circuits," in "Transistors I," RCA Labs., Princeton, N. J., pp. 431-457; 1956.

[12] G. Meyer-Brötz, "Die Vierpolparameter des Flächentransistors in den drei Grundschaltungen," *Telefunken-Zeitung*, vol. 29, pp. 21-28; March, 1956.



(a)

(b)

(c)

Fig. 6—Measurements of relative third harmonics in grounded emitter connection. Vertical scale: Ratio of third to first harmonic. Horizontal scale: dc collector bias current. (a) Transistor type 2 N 114, (b) Transistor type 2 N 113, (c) Transistor type 2 N 137. Full curves are measured; broken curves are calculated. In all cases the input signal is $V_{eb} = 10\ mV$ rms. The collector bias voltage is $V_{CE} = -3$ volts. The curves with the higher $R_{bb'}$ values are obtained by inserting an extra resistance of 100 ohms into the base lead.

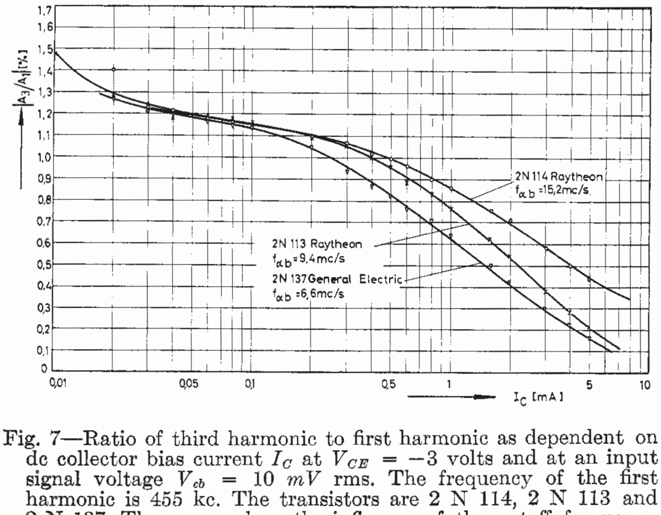


Fig. 7—Ratio of third harmonic to first harmonic as dependent on dc collector bias current I_C at $V_{CE} = -3$ volts and at an input signal voltage $V_{eb} = 10\ mV$ rms. The frequency of the first harmonic is 455 kc. The transistors are 2 N 114, 2 N 113 and 2 N 137. The curves show the influence of the cutoff frequency $f_{\alpha b}$, which is equal to 15. 2 mc for type 2 N 114, equal to 9.4 mc for type 2 N 113 and equal to 6.6 mc for type 2 N 137.

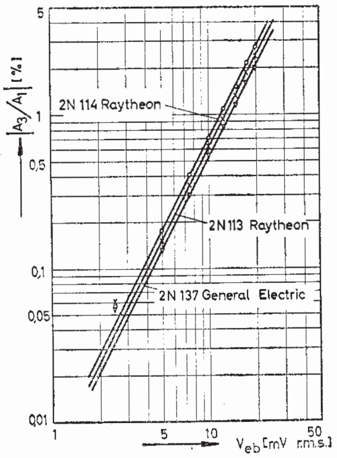
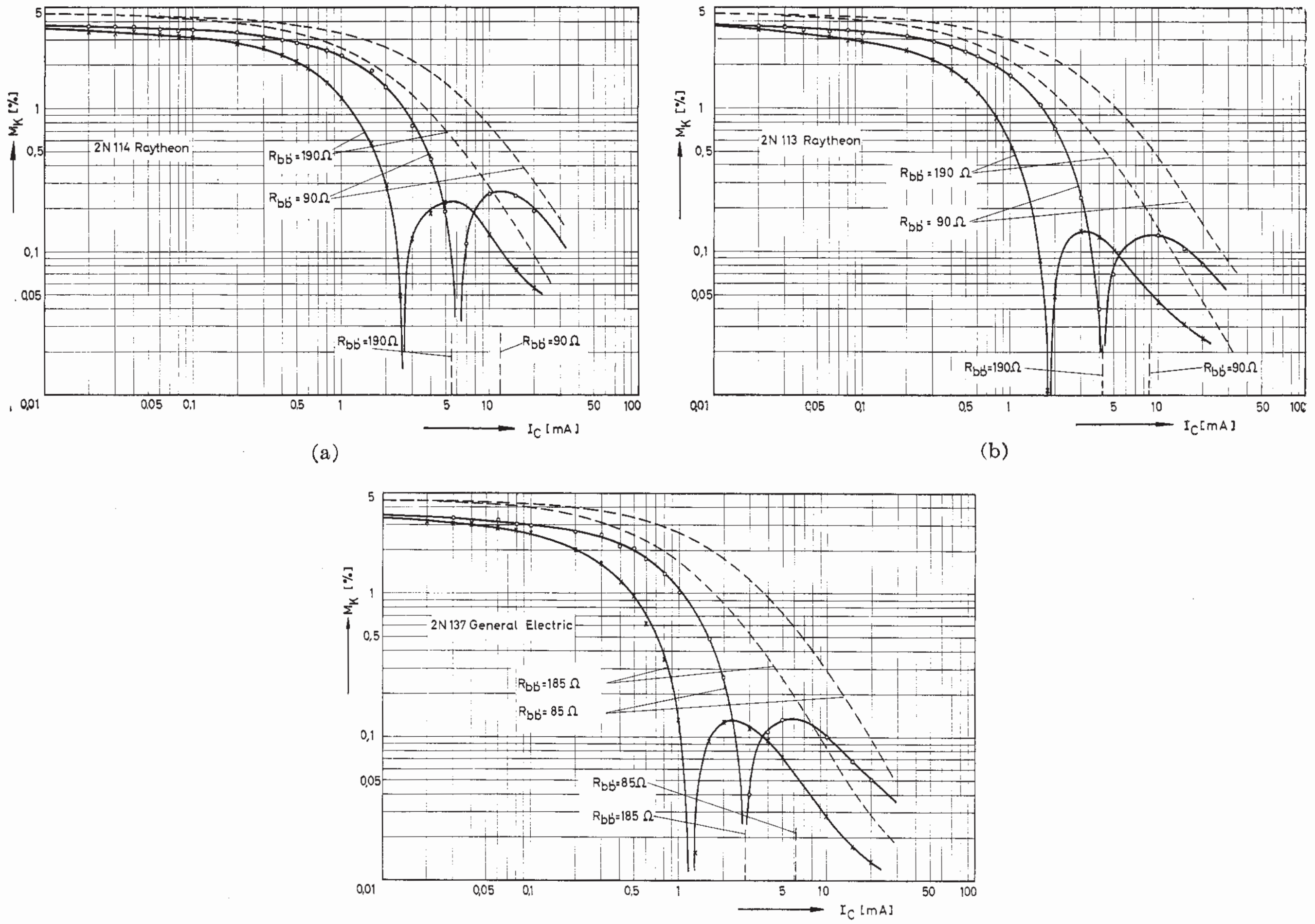


Fig. 8—Ratio of third harmonic to first harmonic at a frequency of the latter of 455 kc, as dependent on rms values of the input signal voltage V_{eb} at a dc collector bias voltage $V_{CE} = -3$ volts and a dc collector bias current I_C of 1 ma. The transistor types are 2 N 114, 2 N 113 and 2 N 137.



(a) (b)

(c)

Fig. 9—Degree M_K of cross modulation as measured (full curves) and calculated (broken curves) in dependence on dc collector bias current I_C at a dc collector bias voltage V_{CE} of −3 volts. The desired signal is of frequency 455 kc and of an effective value $V_0 = 5\ mV$ rms. The spurious signal is of frequency 500 kc and of an effective value $V_s = 10\ mV$ rms, at a degree of amplitude modulation $M = 30$ per cent with a modulation frequency of 400 cps. The dotted vertical lines indicate the calculated positions of the minimum cross modulation values. The base lead resistances $R_{bb'}$ were increased by the addition of an extra 100 ohm. The transistor types are 2 N 114, 2 N 113 and 2 N 137

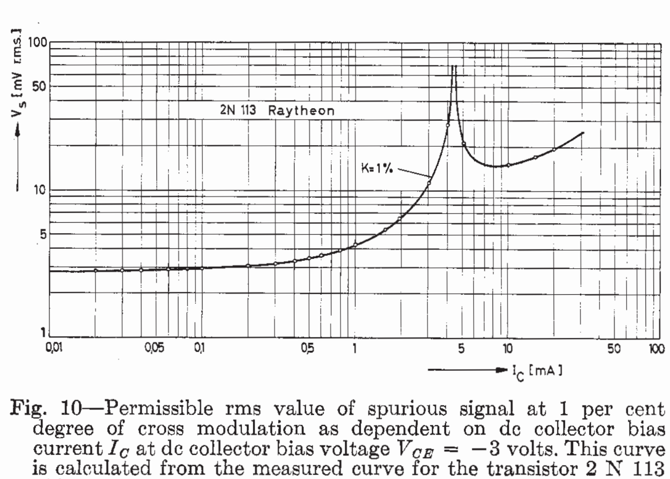


Fig. 10—Permissible rms value of spurious signal at 1 per cent degree of cross modulation as dependent on dc collector bias current I_C at dc collector bias voltage $V_{CE} = -3$ volts. This curve is calculated from the measured curve for the transistor 2 N 113 of Fig. 9.

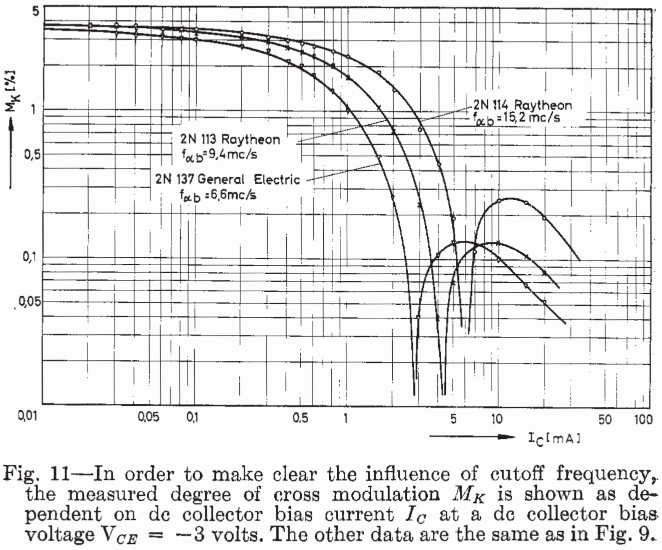


Fig. 11—In order to make clear the influence of cutoff frequency, the measured degree of cross modulation M_K is shown as dependent on dc collector bias current I_C at a dc collector bias voltage $V_{CE} = -3$ volts. The other data are the same as in Fig. 9.

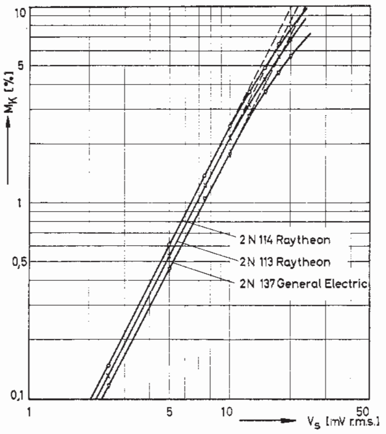


Fig. 12—Measured values of degree M_K of cross modulation as dependent on rms value of the spurious signal at a dc collector bias current I_C = 0.5 ma and at a dc collector bias voltage $V_{CE} = -3$ volts. The desired signal at 455 kc is 5 mV rms and the the spurious signal is at 500 kc, modulated 30 per cent with a modulation frequency of 400 cps. The transistors are 2 N 114, 2 N 113 and 2 N 137.

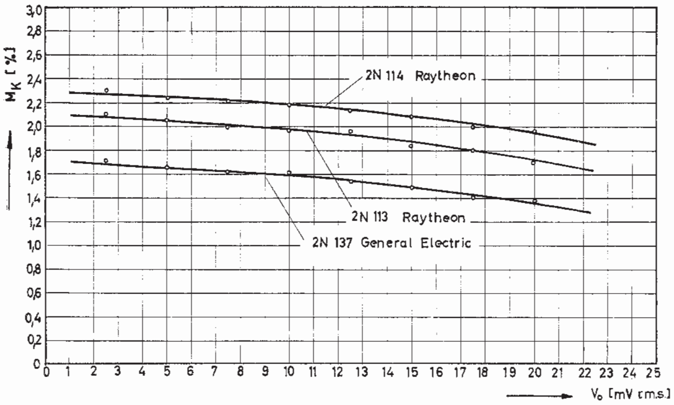


Fig. 13—Measured values of degree M_K of cross modulation as dependent on rms value of the desired signal of 455 kc, at a dc collector bias current I_C = 0.5 ma and a dc collector bias voltage $V_{CE} = -3$ volts. The spurious signal is of frequency 500 kc at 10 mV rms and at a modulation of 30 per cent with a modulation frequency of 400 cps. The transistors are 2 N 114, 2 N 113, and 2 N 137.

The measured curves show a zero value of cross modulation at a definite dc collector current. This is quite unexpected and, as far as the authors know, is shown here for the first time. The situation of the zero point is dependent on $R_{bb'}$. This zero value is not shown by the calculated curves. However, the general course of the measured curves is matched satisfactorily by the calculations. An explanation of the zero points is given in Section V.

With electron tubes it is customary to draw curves, giving the amplitude of the 100 per cent modulated spurious signal, corresponding to a given degree (*e.g.*, 1 per cent) of cross modulation as dependent on tube current or on transconductance. Fig. 10 shows a corresponding curve for the transistor type 2 N 113. This curve has been evaluated from the measurements of Fig. 9. From Fig. 10 it is seen that the spurious signal amplitudes for 1 per cent cross modulation are about 100 times smaller with transistors than with variable-mu tubes. Of course, in the vicinity of the infinite value of Fig. 10, the permissible spurious signal is much larger and may be comparable to that of variable-mu tubes. It has been pointed out by D. D. Holmes[13] that transistor amplifiers with automatic volume control may be made comparable to tube amplifiers with respect to cross modulation performance if special precautions are taken.

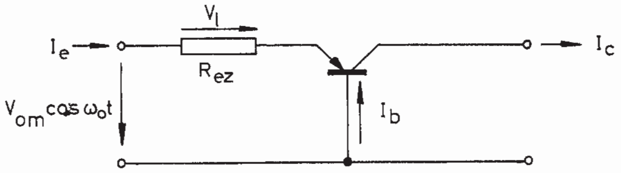


Fig. 14—Our choice of positive directions of ac currents and voltages at a grounded base circuit, including an extra emitter lead resistance R_{ez}.

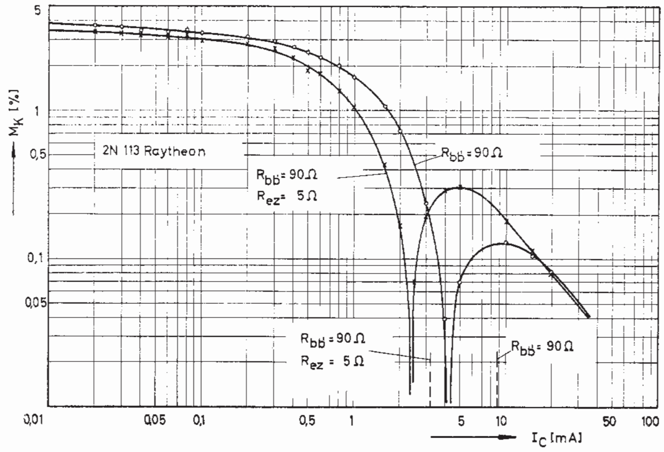


Fig. 15—The influence of an extra emitter lead resistance of 5 ohms, bypassed by 8 microfarads on the position of the minimum value of the cross modulation vs dc collector bias current curve. The transistor is 2 N 113. The dc collector bias voltage $V_{CE} = -3$ volts. The desired signal at 455 kc is 5 mV rms. The spurious signal at 500 kc is 10 mV rms at a modulation of 30 per cent with a modulation frequency of 400 cps.

Fig. 11 shows the dependence of cross modulation on the collector current for transistors with different cutoff frequencies.

In Fig. 12, cross modulation is shown as dependent on spurious signal amplitude.

In Fig. 13, cross modulation is shown as dependent on desired signal amplitude. By theory, the former dependence should be quadratic, and in the latter case, cross modulation should be independent of signal amplitude. This is confirmed satisfactorily by Figs. 12 and 13.

V. EXPLANATION OF THE ZERO CROSS MODULATION AT A DEFINITE COLLECTOR CURRENT.

The said zero cross modulation is due to a counterphase modulation, caused by AF feedback in the transistor circuit under consideration. This possibility was suggested to us by Dr. R. Cantz of Telefunken Company, Ulm, Germany, in a private communication. It has been shown by subsequent research that this is indeed the correct explanation. By the nonlinearity of the transistor the amplitude-modulated RF signal

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[13] D. D. Holmes, "Cross modulation in radio frequency amplifiers," in "Transistors I," RCA Lab., Princeton, N. J., 422–430; March, 1956.

$$v_s = V_{sm}(1 + M\cos\omega_1 t)\cos\omega_s t$$

is detected and the following AF currents ensue in the grounded base connection:

$$\mathbf{I}_e = I_E \frac{\Lambda^2}{2!} V_{sm}^2 M \exp(j\omega_1 t) \qquad (30)$$

$$\mathbf{I}_c = I_C \frac{\Lambda^2}{2!} V_{sm}^2 M \exp(j\omega_1 t). \qquad (31)$$

In the presence of a resistance R_{ez} in the emitter lead (Fig. 14), these currents cause an AF signal which modulates the desired signal. The ensuing collector ac current in the grounded emitter circuit, after some calculations, works out as

$$i_c = -\gamma I_E \Lambda V_{0m} \cdot \left[1 - \frac{\Lambda^3}{2} V_{sm}^2 M I_E R_{ez} \cos\omega_1 t\right] \cos\omega_0 t. \qquad (32)$$

It appears that the sign of this modulation is opposite to that of the cross modulation, considered previously. Hence, both modulations may cancel out. Complete cancellation results if the following condition is satisfied:

$$\Lambda I_E R_{ez} = 1. \qquad (33)$$

A completely analogous effect may be obtained by the insertion of a resistance into the base lead of a grounded emitter circuit. If a resistance R_{ez} is inserted into the emitter lead and at the same time a resistance R_{bz} into the base lead, the condition for complete cancellation of cross modulation becomes

$$\Lambda[(I_E - I_C)R_{bz} + I_E R_{ez}] = 1. \qquad (34)$$

By this calculation the zero point of the cross modulation curves, as well as the shift in position of this zero point by variation of the base lead resistance, is completely understood. The lack of quantitative coincidence between the zero positions as calculated and measured must be ascribed to the several simplifications and neglections in the calculations.

Fig. 15 shows the effect of a resistance of 5 ohms, bypassed by 9 microfarads, on the cross modulation curve. As is to be expected, the resistance, which is solely bypassed at RF, affects only the position of the zero point. Obviously, no RF feedback is caused by this resistance and therefore the two cross modulation curves nearly coincide, except in the vicinity of their zeros.

A similar effect to the cancellation of cross modulation considered here has been found in tube circuits.[14] In such circuits the compensating resistance has to be inserted into the cathode lead.

APPENDIX

DERIVATION OF (2)

The boundary condition for the hole density p at the boundary x_{ne} of the depletion layer between emitter and base, indicated as $p(x_{ne})$, is given, according to Boltzmann's law, as

$$p = p(x_{ne}) = p_n \exp(\Lambda v_{E'B'}). \qquad (35)$$

The voltage $v_{E'B'}$ consists of a dc voltage, which determines the bias position, and of a superimposed ac voltage

$$v_{E'B'} = V_{E'B'} + V_{e'b'm}\cos\omega t. \qquad (36)$$

This combination of voltage is inserted into (35). Thereupon the exponential function is separated into two exponential functions, each containing only one voltage. Then,

$$p = p(x_{ne}) = p_n \exp(\Lambda V_{E'B'}) \exp(\Lambda V_{e'b'm}\cos\omega t). \qquad (37)$$

The second exponential function of this expression is developed into a power series, leaving out powers higher than the third,

$$p = p(x_{ne}) = p_n \exp(\Lambda V_{E'B'}) \cdot \left[1 + \Lambda V_{e'b'm}\cos\omega t + \frac{\Lambda^2}{2!} V_{e'b'm}^2 \cos^2\omega t + \frac{\Lambda^3}{3!} V_{e'b'm}^3 \cos^3\omega t\right].$$

The trigonometric terms of powers beyond the first one are developed into components:

$$p = p(x_{ne}) = p_n \exp(\Lambda V_{E'B'})\left[1 + \frac{1}{2}\frac{\Lambda^2 V_{e'b'm}^2}{2!} + V_{e'b'm}\left(1 + \frac{3}{4}\frac{\Lambda^2 V_{e'b'm}^2}{3!}\right)\cos\omega t + \frac{1}{2}\frac{\Lambda^2 V_{e'b'm}^2}{2!}\cos 2\omega t + \frac{1}{4}\frac{\Lambda^3 V_{e'b'm}^3}{3!}\cos 3\omega t\right]. \qquad (38)$$

In (38) we may now apply the usual complex notation to the trigonometric terms. Then (2) of Section I is obtained.

ACKNOWLEDGMENT

This research was supported by a stipend to the first author from the Swiss Federation under the Bilateral Technical Aid Program for economically underdeveloped countries. A further stipend to the first author was given by the Battelle Memorial Institute at Geneva, Switzerland.

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[14] E. Hudec, "Kreuzmodulation und Eingangsrauschspannung," *Elektr. Nachr. Techn.*, vol. 20, pp. 123–135; May, 1943.